UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the October/November 2007 question paper

4037 ADDITIONAL MATHEMATICS

4037/01   Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the October/November 2007 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M   Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A   Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B   Accuracy mark for a correct result or statement independent of method marks.

•   When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

•   The symbol $\sqrt{}$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

•   Note: B2 or A2 means that the candidate can earn 2 or 0.
              B2, 1, 0 means that the candidate can earn anything from 0 to 2.

The following abbreviations may be used in a mark scheme or used on the scripts:

AG   Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD   Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO   Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

ISW   Ignore Subsequent Working

MR   Misread

PA   Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS   See Other Solution (the candidate makes a better attempt at the same question)

Penalties

MR -1   A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{}$" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

OW -1,2 This is deducted from A or B marks when essential working is omitted.

PA -1   This is deducted from A or B marks in the case of premature approximation.

S -1   Occasionally used for persistent slackness – usually discussed at a meeting.

EX -1   Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

1 $A = \begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$, $A^2 = \begin{pmatrix} 1 & -3 \\ 9 & -2 \end{pmatrix}$

$\begin{pmatrix} 1 & -3 \\ 9 & -2 \end{pmatrix} + m\begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix} = n\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

	M1	Reasonable attempt (needs 2 correct)
	A1	All correct
$1 + 2m = n$ and $-3-m=0$	M1	Identity matrix must be correct
→ $m = -3$ and $n = -5$	A1	Equating their elements once
	[4]	co.

2 $\left(\dfrac{1}{1-\cos\theta} - \dfrac{1}{1+\cos\theta}\right) \equiv 2\,\mathrm{cosec}\,\theta\cot\theta$

Manipulation of fractions	M1	$(1-c)(1+c)$ in denominator +reasonable attempt at numerator (ignore signs)
$(1-c)(1+c) = s^2$ used	B1	
$\dfrac{2\cos\theta}{\sin^2\theta} \Rightarrow 2\mathrm{cosec}\theta\cot\theta$	M1	Knowledge of cot and cosec
All correct	A1	When all correct
	[4]	a.g Beware fortuitous answers.

3 (i) $p = \dfrac{\sqrt{3}+1}{\sqrt{3}-1} \rightarrow p = \dfrac{\sqrt{3}+1}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1}$

	M1	× top and bottom by √3 + 1
	A1	Denominator = 2
→ $p = \dfrac{3+2\sqrt{3}+1}{3-1} = 2 + \sqrt{3}$	A1 [3]	co
(ii) either $p - \dfrac{1}{p} = 2 + \sqrt{3} - \dfrac{1}{2+\sqrt{3}}$		
or $p - \dfrac{1}{p} = \dfrac{p^2-1}{p}$	M1 A1 [2]	Complete method. co.
→ $2\sqrt{3}$		

4 (i) 4 men from 9 = $_9C_4$ (126)

4 women from 6 = $_6C_4$ (15)	B1	For either $_9C_4$ or $_6C_4$
Multiply together → 1890	M1 A1 [3]	Product of 2 $_nC_r$s. co
(ii) One twin included $(_7C_3 \times _6C_4)$	M1	For 2 $_nC_r$s.
To include other twin ×2	DM1	Two times his first answer.
→ 1050	A1 [3]	co

5 (i) Resultant vel = $(960\mathbf{i} + 400\mathbf{j}) \div 4$

→ $(240\mathbf{i} + 100\mathbf{j})$	M1 A1	Division of distance by time co (could be wind × 4) then ÷ 4 later
$\mathbf{v}$ (still air) = $(240\mathbf{i} + 100\mathbf{j}) - \mathbf{wind}$	M1	Needs subtraction
= $300\mathbf{i} + 40\mathbf{j}$	A1 [4]	co
(ii) $\tan\theta = 40 \div 300$ (→ 7.6º)	M1	Use of tan with their 2 components Not $960\mathbf{i} + 400\mathbf{j}$
→ Bearing of 082º (awrt 82º)	A1 [2]	

6 (i) $\dfrac{dy}{dx} = \dfrac{6}{\sqrt{4x+1}}$

$y = \dfrac{6(4x+1)^{\frac{1}{2}}}{\frac{1}{2}} \div 4 \qquad (+c)$

Uses (6,20) $\to c = 5$

$(y = 3\sqrt{4x+1} + 5)$

B1	For an expression involving $(4x+1)^{\frac{1}{2}}$
B1	For all correct
M1	Uses (6,20) in an integration involving
A1	$(4x+1)^{\frac{1}{k}}, k \neq -\dfrac{1}{2}$
[4]	co (do not mark after +5)

(ii) Perp to −½ = 2

$\dfrac{6}{\sqrt{4x+1}} = 2 \quad \to \quad x = 2, y = 14$

Eqn $\to y - 14 = -½(x - 2)$ or $2y+x=30$
$\to$ (0, 15) and (30, 0)

M1	Use of $m_1 m_2 = -1$, with attempt to solve co on y-value, using $x = 2$
A1√	
M1	Correct method for line
A1	co
[4]	

7 (i) $2^{2x} = 2^{x+2} + 5$

$2^{2x} = u^2 \qquad 2^{x+2} = 4u$
Solution of quadratic $u^2 = 4u+5$
$2^x = 5 \quad \to \quad x = \lg 5 \div \lg 2$
$\to \quad x = 2.32$

B1 B1	co co
M1	Correct method of solution of quad=0
M1	From $2^x = k$ to x by correct method
A1	co – loses if more than one answer
[5]	given.

(ii) $2\log_9 3 + \log_5(7y - 3) = \log_2 8$.

$2 \times ½ + \dots = 3$
$\log_5(7y - 3) = 2$
$(7y - 3) = 25 \quad \to \quad y = 4$

B1	For ½
B1	For RHS = 3
M1 A1	From $\log_5$ to $5^p = k$. co
[4]	

8 (a) $f(1) = 1-11+k-30 \to k - 40$
$f(2) = 8-44+2k-30 \to 2k - 66$
$f(1) = 4f(2)$
$\to k = 32$

M1	Uses either $x = 1$ or 2, not −1 or −2.
A1	Both correct, unsimplified.
M1	Linked + solution – allow if 4 on LHS
A1	co
[4]	

(b) $x^3 - 4x^2 - 8x + 8 = 0$
Tries for a first solution $\to x = -2$
Divides by $(x -$ his first solution)
$\to x^2 - 6x + 4 = 0$
$\to x = \dfrac{6 \pm \sqrt{20}}{2} \to 3 \pm \sqrt{5}$

M1 A1	Search shown for M,$x = -2$ gets M1A1.
M1	Correct method.
DM1	Correct method for soln of quadratic
A1	Must be simplified.
[5]	

9 (i)

x	2	4	6	8	10
y	14.4	10.8	11.2	12.6	14.4
xy	29	43	67	101	144
x^2	4	16	36	64	100

M1 — Knows what to do.
A1 [2] — Mark from graph – 5 points are in line.

(ii) Gradient 1.2 (±0.1) — B1 — co
'y' intercept (± 2) — B1 — co

$\rightarrow y = 1.2x + \dfrac{24}{x}$

M1 — $xy = $ (their grad)x + (their intercept)
A1 [4]

(iii) From graph $xy = 83 \rightarrow x^2 = 49$ — M1 — Reads on vertical axis at 83
Valid method to obtain y — M1 — Valid method to obtain y
$y = 11.6 – 12.2$ — A1 [3] — co

10 (i) $BC = 2(10\sin 0.4) = 7.79$ — M1 A1 [2] — Any correct method – cos rule ok.

(ii) $\angle ABC = \frac{1}{2}(\pi - 0.8) = 1.17$ rads — B1 — Anywhere in the question.
Arc $CD = 7.79 \times 1.17$, Arc $BC = 10 \times 0.8$ — M1 — Use of $s = r\theta$ in either arc.
$\rightarrow P = $ sum of the arcs + BD (=7.79) — M1 — Overall plan – arc CD + arc BC + BD
$\rightarrow P = 24.9$ — A1 [4] — co.

(iii) Area sector $BDC = \frac{1}{2}(7.79)^2 \times 1.17$ — M1 — Use of $A = \frac{1}{2}r^2\theta$ for sector BDC
Area segment on $BC = \frac{1}{2}.10^2(0.8 - \sin 0.8)$ — B1 — B1 for $0.5(10)^2 \, 0.8$

B1 — B1 for $0.5(10)^2 \sin 0.8$

$\rightarrow$ Shaded area = 39.6 or 39.7 — A1 [4] — co

11 EITHER

(i) $y = xe^{2x}$ d/dx(e^{2x}) = $2e^{2x}$ — B1 — Anywhere – even if product not used
$\rightarrow$ dy/dx = $e^{2x} + 2x\, e^{2x}$ — M1A1 — Use of correct formula for "uv". co

$\rightarrow$ d^2y/dx^2 = $2e^{2x} + 2e^{2x} + 4xe^{2x}$ — M1A1 [5] — Use of product formula again. co.

(ii) dy/dx = 0 when $1 + 2x = 0 \rightarrow x = -\frac{1}{2}$ — M1 A1 — Sets his dy/dx to 0 and tries to solve.

$\rightarrow y = -\frac{1}{2}e^{-1} = -\dfrac{1}{2e}$. — A1 [3] — co – ag – beware fortuitous results.

(iii) If $x = -\frac{1}{2} \rightarrow$ +ve result — M1 — Looks at sign.
$\rightarrow$ Minimum — A1 — Correct deduction from correct x.
(or gradient goes $-, 0, +$) — (or by any other valid method)

(or y value to left or right of $(-\frac{1}{2})$ $> -\dfrac{1}{2e}$) [2]

11 OR

(i) d/dx(lnx) = 1/x

$$\frac{d}{dx}\left(\frac{\ln x}{x^2}\right) = \frac{x - 2x\ln x}{x^4} = \frac{1 - 2\ln x}{x^3}$$

B1	Anywhere – even if quotient not used
M1	Use of correct quotient formula
A1	co
[3]	

(ii) dy/dx = 0 → lnx = ½ → x = √e

$$\to y = \ln(\sqrt{e}) \div e = \frac{1}{2e}.$$

M1 A1	Sets his dy/dx to 0 and tries to solve.
A1	co – ag – beware fortuitous results.
[3]	

(iii) $\dfrac{\ln x}{x^2} = \int\left(\dfrac{1}{x^3}\right)dx - \int\dfrac{2\ln x}{x^3}dx$

$$\int\frac{\ln x}{x^3}dx = \frac{1}{2} \times [\int\left(\frac{1}{x^3}\right)dx - \frac{\ln x}{x^2}]$$

$$\to \quad = \frac{1}{2}\left(\frac{x^{-2}}{-2} - \frac{\ln x}{x^2}\right) + c$$

M1	Recognition that integration is the reverse of differentiation.
B1	B1 for ½.
B1	B1 for $(x^{-2}) \div (-2)$
A1	All ok including +c.
[4]	

DM1 for quadratic equation. Equation must be set to 0 if using formula or factors.

Formula.	Factors
Must be correct	Must attempt to put quadratic into 2 factors.
– ignore arithmetic and algebraic slips.	Each factor then equated to 0.